





NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION

CLASS - 11 (PCM)

Question Paper Code : UN497

KEY

1. C	2. A	3. A	4. A	5. A	6. D	7. B	8. A	9. B	10. C
11. B	12. C	13. D	14. D	15. A	16. C	17. D	18. B	19. B	20. C
21. A	22. A	23. C	24. D	25. A	26. D	27. C	28. D	29. D	30. C
31. D	32. C	33. A	34. A	35. B	36. C	37. B	38. B	39. A	40. B
41. C	42. C	43. C	44. A	45. D	46. C	47. D	48. A	49. C	50. A
51. B	52. B	53. A	54. B	55. A	56. A	57. A	58. B	59. B	60. A

SOLUTIONS

MATHEMATICS

01. (C) There are a total of 2^{100} subsets.

The number of subsets containing 0, 1, 2, 3, \ldots 49 elements is exactly the same as those containing 100, 99, 98, 97, \ldots 51 elements.

Since the number of subsets with 50 elements is ${}^{100}C_{50}$, the answer is:

$$\frac{2^{100} - {}^{100} C_{50}}{2} + {}^{100}C_{50} = 2^{99} + \frac{{}^{100}C_{50}}{2}$$

02. (A) Let N be the number of one – one functions from {1, 2, 3} into {a, b, c}.

$$\Rightarrow$$
 N = ${}^{3}P_{3}$

03. (A) Let 'r' be the common ratio Then

> $x_{2} = x_{1}r, x_{3} = x_{2}r = x_{1}r^{2}, y_{2} = y_{1}r, y_{3} = y_{1}r^{2}$ Q = $(x_{2}, y_{2}) = (x_{1}r, y_{1}r)$ R = $(x_{3}, y_{3}) (x_{1}r^{2}, y_{1}r^{2})$ We observed that PQ + QR = PR

... P, Q, R are collinear.

04. (A) The equation represented by 08. (A) $Z\overline{Z} + a\overline{Z} + a\overline{Z} + b = 0$ is a circle centered at "-a" and radius $\sqrt{|a|^2 - b}$. The radius of given circle ... $=\sqrt{25-5}=\sqrt{20}$ $=2\sqrt{5}$ 09. (B) 05. (A) Each of the three persons can leave the lift at one of the ten floors (other than 2nd storey and one at which they enter the lift). Since they leave the lift at different storeys. 10. (C) Number of ways = ${}^{10}P_{3} = 10 \times 9 \times 8 = 720$ $(1 + x^2)^5 \times (1 + x)^4$ 06. (D) 11. (B) $= (1 + 5x^{2} + 10x^{4} + 10^{6} + 5x^{8} + x^{10}) \times$ $(1 + 4x + 6x^2 + 4x^3 + x^4)$ $= \dots + (5x^2)(4x^3) + (10x^4)(4x) + \dots$ Hence the coefficient of $x^5 = 20 + 40 = 60$ 07. (B) Letter the GP be a, ar, ar²,...., where 0 < r < 1. Then, $a + ar + ar^{2} + ... = 3$ and $a^{2} + a^{2}r^{2} + a^{2}r^{4} + \dots = \frac{9}{2}$ $\Rightarrow \frac{a}{1-r} = 3 \text{ and } \frac{a^2}{1-r^2} = \frac{9}{2}$ $\Rightarrow \frac{9(1-r)^2}{1-r^2} = \frac{9}{2}$ 12. (C) $\Rightarrow \frac{1-r}{1+r} = \frac{1}{2} \Rightarrow r = \frac{1}{3}$ Putting $r = \frac{1}{3}$ in $\frac{a}{1-r} = 3$, we get a = 2Now, the required sum of the cubes is $a^{3} + a^{3}r^{3} + a^{3}r^{6} + \dots = \frac{a^{3}}{1 - r^{3}}$... $=\frac{8}{1-(1/27)}=\frac{108}{13}$

Here, $|F_1F_2| = \sqrt{(3-0)^2 + (4-0)^2} = 5$ \Rightarrow | PF₁ | + | PF₂ | = 10 > 5 \Rightarrow | PF₁ | + | PF₂ | = a constant greater than $|F_1F_2|$, therefore, locus of P is an ellipse with foci at F₁ and F₂. Length of the diagonal of the square $=\sqrt{(1-2)^{2}+(-2+3)^{2}+(3-5)^{2}}=\sqrt{6}$ \therefore Length of its side = $\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$ Required number = $8 \times 9 \times 9$ $= 8 \times 81 = 648$ Normal at (at², 2at) on the parabola y^2 = 4ax is $y + tx = 2at + at^{3}$ Suppose normal equation (i) cuts the curve again at $(at_1^2, 2at_1)$, then $2at_1 + att_1^2 = 2at + at^3$ \Rightarrow 2a(t - t¹) + at (t² - t₁²) = 0 or $2 + t(t + t_1) = 0$ $(\therefore a(t-t_1) \neq 0)$ $\therefore t_1 = -t - \frac{2}{t}$ $=-\left(t+\frac{2}{t}\right)$ Equation of the tangent at point ' θ ' is $\frac{x}{2}\cos\theta + \frac{y}{2}\sin\theta = 1$ (1)

Given line is, $\frac{x}{a} \cdot \frac{1}{\sqrt{2}} + \frac{y}{b} \cdot \frac{1}{\sqrt{2}} = 1$ (2)

Since (2) touches the ellipse (1),

we have $\cos \theta = \frac{1}{\sqrt{2}}$ and $\sin \theta = \frac{1}{\sqrt{2}}$ $\theta = 45^{\circ}$

13. (D)
$$f(x) = \frac{3}{4-x^2} + \log_{10} (x^3 - x)$$

For domain of $f(x)$
 $x^3 - x > 0$
 $\Rightarrow x (x - 1) (x + 1) > 0$
 $\Rightarrow the region is (-1, 0) \cup (1, \infty)$
Also, $4 - x^2 \neq 0 \Rightarrow x \neq \pm 2$
 \Rightarrow the region is (-0, 2) $\cup (2, \infty)$
 \therefore Common region is (-1, 0) $\cup (1, 2) \cup (2, \infty)$
14. (D) $\lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x} = \lim_{x\to 0} \frac{|\sin x|}{x}$
 $LHL = \lim_{x\to 0^+} \frac{|\sin x|}{x} = \lim_{h\to 0} \frac{|\sin (-h)|}{-h}$
 $= \lim_{h\to 0} \frac{\sin h}{-h} = -1$
RHL = $\lim_{x\to 0^+} \frac{|\sin x|}{x} = \lim_{h\to 0} \frac{|\sin h|}{h} = \lim_{h\to 0} \frac{\sin h}{h} = 1$
Since LHL \neq RHL,
 $\therefore \lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x}$ does not exist
15. (A) The given equation is
 $(c^2 - ab) x^2 - 2 (a^2 - bc) x + (b^2 - ac) = 0$
Since the roots are equal, $B^2 - 4AC = 0$
 $\Rightarrow 4(a^2 - bc)^2 - 4 (c^2 - ab) (b^2 - ac) = 0$
 $\Rightarrow [a^4 - 2a^2bc + b^2c^2] - [b^2c^2 - ab^3 - ac^3 + a^2bc] = 0$
 $\Rightarrow a(a^3 + b^3 + c^3 - 3abc) = 0$
 \therefore Either $a = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$
16. (C) The probability that the problem is not solved
 $= (1 - \frac{1}{2})(1 - \frac{1}{3})(1 - \frac{1}{4}) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$
 \therefore The probability that problem is solved
 $= 1 - \frac{1}{4} = \frac{3}{4}$

(D)
$$y = \left\{x + \sqrt{x^2 - 1}\right\}^{15} + \left\{x - \sqrt{x^2 - 1}\right\}^{15}$$

Differentiate w.r.t 'x'

$$\frac{dy}{dx} = 15\left(x + \sqrt{x^2 - 1}\right)^{14}\left(1 + \frac{x}{\sqrt{x^2 - 1}}\right) + 15\left(\left(x - \sqrt{x^2 - 1}^{14}\right)\left(1 - \frac{x}{\sqrt{x^2 - 1}}\right)\right)$$

$$\frac{dy}{dx} = \frac{15}{\sqrt{x^2 - 1}} \cdot y$$

$$\sqrt{x^2 - 1} \cdot \frac{dy}{dx} = 15 y$$
Again differentiating both sides w.r.t x

$$\frac{x}{\sqrt{x^2 - 1}} \cdot \frac{dy}{dx} + \sqrt{x^2 - 1} \frac{d^2 y}{dx^2} = 15 \frac{dy}{dx}$$

$$x \frac{dy}{dx} + \left(x^2 - 1\frac{d^2 y}{dx^2}\right)$$

$$= 15\sqrt{x^2 - 1} \cdot \frac{15}{\sqrt{x^2 - 1}} \cdot y = 225y$$
(B)
$$f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$$

$$f'(x) = \frac{1}{2\sqrt{ax}} \cdot a + a^2 \left[-\frac{1}{2}(ax)^{-\frac{3}{2}}a\right]$$

$$f'(a) = \frac{1}{2} - \frac{a^3 \cdot a^{-3}}{2} = 0$$
(B) We have, $f(x) = \sin x - \cos x$

$$x = \sqrt{2} \sin\left(x - \frac{\pi}{4}\right)$$
Clearly, $f(x)$ is defined for all real x
 \therefore Domain of $f = (-\infty, \infty)$
Let $y = \sqrt{2} \sin\left(x - \frac{\pi}{4}\right)$
 $\Rightarrow x - \frac{\pi}{4} = \sin^{-1} \frac{y}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4} + \sin^{-1} \frac{y}{\sqrt{2}}$
For x to be real,

$$-1 \le \frac{y}{\sqrt{2}} \le 1 \Rightarrow -\sqrt{2} \le y \le \sqrt{2}$$
 \therefore Range of $y = \left[-\sqrt{2}, \sqrt{2}\right]$

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17.

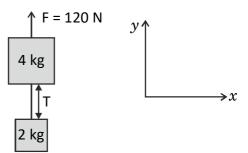
18.

25. (A) P(n) = n2 - n + 41

- \Rightarrow P(3) = 9 3 + 41 = 47 (prime)
- & P(5) = 25 5 + 41 = 61 (prime)
- ... P(3) and P(5) are both prime i.e., true.

PHYSICS

- 26. (D) During upward motion, the gravity effect and air resistance will oppose its motion. Due to which the speed of the body decreases and becomes zero at the highest point. Afterwards the body moves downwards due to gravity pull but air resistance opposes. Still the velocity of body increases.
- 27. (C) Let 'a' be the acceleration of the blocks and T the tension in the string as shown below.



Taking the two blocks and the string as the system,

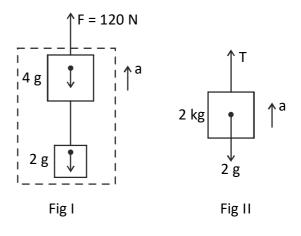
Using
$$\Sigma F_{y} = ma_{y}$$
, we get

F - 4g - 2g = (4 + 2)a

60 = 6 a

 $a = 10 \text{ m/s}^2$

Diagram of 2 kg block is as shown below in fig II



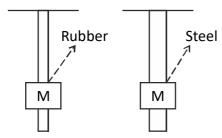
Using $\Sigma F_y = ma_y$, we get

T – 2g = 2a

or T – 20 = (2) (10) T = 40 N

...

28. (D) According to the diagram shown below in which a mass M is attached at the centre of each rod, then both the rods will be elongated. But due to different elastic properties of material, the steel rod will elongate without making any perceptible change in shape, but the rubber rod will elongate with the shape of the bottom edge tapered to a tip at the centre.



29. (D) Solid and hollow balls can be distinguished by any one of the three methods. $I_h > I_s$. When torques are equal, angular acc. a of hollow ball must be smaller than a of solid ball.

The velocity of a rolling body is inversely proportional to the moment of inertia. As the moment of inertia for a hollow sphere is more than that of a solid sphere, the solid sphere rolls down faster than a hollow sphere.

Similarly, on rolling, solid ball will reach the bottom before the hollow ball.

30. (C) SI unit of energy is $u_1 = \left[M_1 L_1^2 T_1^{-2}\right]$ Now, $M_2 = 1 \text{ kg}$, $T_2 = 1 \text{ minute} = 60 \text{ s}$, $L_2 T_2^{-2} = 10 \text{ ms}^{-2}$ From $L_2 T_2^{-2} = 10$ $L_2 = 10 T_2^2 = 10(60)^2 = 36000 \text{ m}$ $u_2 = M_2^1 L_2^2 T_2^{-2}$ $= (1 \text{ kg})^1 (36000 \text{ m})^2 (60 \text{ s})^{-2}$ $= 0.36 \times 10^6 \text{ J}$ 31. (D) As there is no component of gravity along the horizontal (x - axis), therefore $a_x = 0$

$$v_x = \text{constant or } \frac{dx}{dt} = \text{constant.}$$

- 32. (C) The gravitational attraction on a body due to the earth decreases with height and increases due to the moon. At a certain height, it becomes zero and with further increase in height, the gravitational attraction of the moon becomes more than that of the earth.
- 33. (A) According to continuity equation (Law of Conservation of mass)

$$a_{1}v_{1} = a_{2}v_{2}$$

$$\frac{v_{1}}{v_{2}} = \frac{a_{2}}{a_{1}} = \frac{\pi \left(\frac{d_{2}}{2}\right)^{2}}{\pi \left(\frac{d_{1}}{2}\right)^{2}} = \frac{d_{2}^{2}}{4} \cdot \frac{4}{d_{1}^{2}} = \frac{d_{2}^{2}}{d_{1}^{2}}$$

$$\frac{v_{1}}{v_{2}} = \frac{(3.75)^{2}}{(2.50)^{2}} = \left[\frac{3}{2}\right]^{2}$$

$$\frac{v_{1}}{v_{2}} = \frac{9}{4}$$
or $v_{1} : v_{2} : 9: 4$

- 34. (A) According to Work Energy theorem the work done by all the forces acting on a particle is equal to the change in its kinetic energy i.e., $W = \Delta K$.
- 35. (B) Maxwell derived equation gives the distribution of molecules in different speeds as follows:

The masses of hydrogen and oxygen molecules are different.

For a function f(v), the number of molecules n = f(v), which are having speeds between v and +dv. The Maxwell-Boltzmann speed distribution function $(N_v = dn/dv depends on the mass of the$ gas molecules.

For each function $f_1(v)$ and $f_2(v)$, n will be different. Hence, each function $f_1(v)$ and $f_2(v)$ will obey the Maxwell's distribution law separately.

36. (C) Vertical velocity after 10 s is

$$v = (98 \sin 60^{\circ}) - (9.8) \times 10 = 98 \times \frac{\sqrt{3}}{2} - 98$$

$$= 98 \left[\frac{\sqrt{3}}{2} - 1 \right] = 98 \times (0.866 - 1)$$

$$= -98 \times 0.134$$
Vertical momentum of the ball after 10
s = m.v

$$= 0.5 \times (-98 \times 0.134) \text{ kg m/s.}$$
Initial vertical momentum of the ball
= mu sin 60°

$$= 0.5 \times 98 \times \frac{\sqrt{3}}{2} = 0.5 \times 98 \times 0.866$$
Change in vertical momentum

$$= 0.5 \times 98 \times 0.866 - (-0.5 \times 98 \times 0.134)$$

$$= 0.5 \times 98 [0.866 + 0.134]$$

$$= 0.5 \times 98 = 49 \text{ kg m/s}$$
Horizontal component velocity remains
the same, so no change in momentum
along horizontal direction
∴ change in momentum = 49 kg m/s
37. (B) Here, T₁ = T, W = 6 R

$$\gamma = \frac{C_p}{C_v} = \frac{5}{3}, T_2 = ?$$
In an adiabatic process,

$$W = \frac{R(T_2 - T_1)}{1 - \gamma}$$

$$6 R = \frac{R(T_2 - T)}{1 - \frac{5}{3}}$$

 $T_2 - T = 6\left(-\frac{2}{3}\right) = -4$

 $T_2 - (T - 4)K$

38. (B) On the surface of the earth $g = \frac{GM}{R^2}$; Weight mg = 99 N At a height h above the earth $g' = \frac{GM}{(R+h)^2}$; where $h = \frac{R}{2}$ $\frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{\left(R+\frac{R}{2}\right)^2} = \frac{R^2}{\frac{9}{4}R^2}$ $g' = \frac{4g}{9}$ Weight $= mg' = m \times \frac{4g}{9} = mg \times \frac{4}{9}$ Here $mg = 99 \text{ N} = 99 \times \frac{4}{9} = 44 \text{ N}$ 39. (A) $F \propto A^a \cup^b D^c$ $[MLT^{-2}] \propto [L^2]^a [LT^{-1}]^b [ML^{-3}]^c$ $[MLT^{-2}] \propto M^c L^{2a+b-3c} T^{-b}$

Equating the powers of M, L and T, we get c = 1, 2a + b - 3c = 1, -b = -2

Solving we get a = 1, b = 2, c = 1

40. (B) Sum of P.E. and K.E. of the ball at A = Sum of P.E. and K.E. of the ball at B. At the point A the ball is at rest, so K.E. = 0.

At B, the speed of the ball is $\upsilon.$

At B, h = 0, So P.E. = 0

(P.E. + K.E.) at A = (P.E. + K.E.) at B

$$mgh + 0 = 0 + \frac{1}{2} mv^{2}$$
$$v = \sqrt{2gh}$$
$$= \sqrt{2 \times 9.8 \times 10}$$
$$= 14 m/s$$

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CHEMISTRY

41.	(C)	The	electronic	configuration	of	
		Gadolinium Gd (Z = 64) is [Xe] $4f^7 5d^1 6$				

42. (C) London dispersion forces operate only over very short distance. The energy of interaction varies as

1

(Distance between two interacting particles)

Large or more complex are the molecules, greater is the magnitude of London forces. This is obviously due to the fact that the large electron clouds are easily distorted or polarised.

Hence, greater the polarisability of the interacting particles, greater is the magnitude of the interaction energy.

43. (C)
$$[Ag^+] = 2.2 \times 10^{-4} \text{ mol } L^{-1}$$

 $[C_2O_4^2] = 0.5 [Ag^+]$

 $K = 5.3 \times 10^{-12}$

=
$$0.5 \times 2.2 \times 10^{-4} \text{ mol } \text{L}^{-1}$$

=
$$1.1 \times 10^{-4} \text{ mol } L^{-1}$$

 $K_{sp} = [Ag^+]^2 [C_2O_4^2]$
 $K_{sp} = (2.2 \times 10^{-4} \text{ mol } L^{-1})^2 \times 1.1 \times 10^{-4} \text{ mol } L^{-1}$

 $\frac{\text{Weight of HNO}_3}{\text{Molecular mass of HNO}_3 \times \text{Volume of solution (in L)}}$

 \therefore Weight of HNO₃

= Molarity × Molecular mass Volume (in L)

$$= 2 \times 63 \times \frac{1}{4} = 31.5 \text{ g}$$

It is the weight of 100% HNO₃

But the given acid is 70% HNO₃

:. Its weight =
$$31.5 \times \frac{100}{70}$$
 g = 45 g

45. (D) On moving down the group from B to TI, a regular decreasing trend in the ionisation energy values is not observed.

Elements	В	Al	Ga	In	ΤI
IE (kJ/mol ⁻¹)	801	577	579	558	589

In Ga, there are ten d-electrons in the penultimate shell which screen the nuclear charge less effectively and thus, outer electron is held firmly. As a result, the ionisation energy of both A*l* and Ga is nearly the same. The increase in ionisation energy from In to TI is due to poor screening effect of 14f electrons present in the inner shell.

The correct order of Ionisation enthalpy of group 13 elements is

B > Al < Ga > In < TI.

- 46. (C) LiC*l* would ionize more in water than NaC*l* is a wrong statement because LiC*l* has covalent character.
- 47. (D) BaCO₃ is most thermally stable as the size of Ba²⁺ is big due to which it is having low polarising power and thus cannot polarize oxygen electrons due to which carbonates of large cations are stable.
- 48. (A) Lonic hydrides in their molten state can conduct electricity and on electrolysis liberate dihydrogen gas at anode.

At anode $2H^- \rightarrow H_2(g) + 2e^-$

- 49. (C) It has maximum surface tension due to maximum intermolecular hydrogen bonding.
- 50. (A) Iodine oxideses $S_2O_3^{2-}$ to $S_4O_6^{2-}$ in which sulphur has low oxidation state of +2.5. But bromine oxideses $S_2O_3^{2-}$ to $S_2O_4^{2-}$ in which sulphur has higher oxidation state of +6.

So, bromine oxidises more and therefore, it is a stronger oxidant than iodine.

51. (B) ΔH = Enthalpy of vapourisation

= 30 kJ/mol = 30000 J/mol

 ΔS = entropy of vaporisation

= 75 J/mol/K

At boiling point, the reversible process of liquid = vapour is in equilibrium at 1 atm pressure.

$$\therefore T = \frac{\Delta H}{\Delta S} = \frac{30000 \text{ J mol}^{-1}}{75 \text{ J mol}^{-1} \text{K}^{-1}} = 400 \text{ K}$$

52. (B) Strength of hydrogen bonding depends on the size and electronegativity of the atom. Smaller the size of the atom, greater is the electronegativity and hence stronger is the hydrogen bonding and also the number of hydrogen bonds.

Thus, the order of strength of H bonding is H.....F > H O > H N.

But each HF molecule is linked only to two other HF molecules while each H_2O molecule is linked to four other H_2O molecules through H-bonding.

Hence, the correct decreasing order of the boiling points of given compounds is $H_2O > HF > NH_3$

53. (A) The blue colour of the solution of alkali metal like sodium dissolved in liquid ammonia is due to the ammoniated electron which absorbs energy in the visible region of light and thus imparts blue colour to the solution.

$$\mathsf{M} + (x + y)\mathsf{NH}_{3} \rightarrow [\mathsf{M}(\mathsf{NH}_{3})_{x}]^{+} + [\mathsf{e}(\mathsf{NH}_{3})_{y}]^{+}$$

54. (B) Frequency of first line in Balmer series is

$$\upsilon = 3.29 \times 10^{15} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] s^{-1}$$

Here,
$$n_1 = 2$$
, $n_2 = 3$
= 3.29 × 10¹⁵

$$\left[\frac{1}{(2)^{2}} - \frac{1}{(3)^{2}}\right]$$

= 3.29 × 10¹⁵
 $\left[\frac{1}{4} - \frac{1}{9}\right]$
= 3.29 × 10¹⁵ × $\frac{5}{36}$

55. (A) 1-phenyl-2-butene shows geometrical isomerism in the form of cis and trans isomers as both the double bonded carbons are differently substituted as given below.

$$\begin{array}{ccc} Ph & CH_3 & Ph & H \\ H & H & H \\ (cis) & (trans) \end{array}$$

CRITICAL THINKING

56. (A) When the automobile is in a tunnel the horn sound will be louder than normal.

58. (B) The passage says the opposite, "When Europeans first reach the Americas they found healers who used many plants to cure illnesses. The Europeans adopted many of these treatments..."

Thursday and Monday

